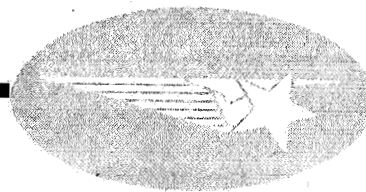


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LUNAR TARGETING STUDY  
INTERIM REPORT  
LUNAR AND PLANETARY  
EPHEMERIS TAPES

January 1972

*Lockheed*

HUNTSVILLE RESEARCH & ENGINEERING CENTER

LOCKHEED MISSILES & SPACE COMPANY

A GROUP DIVISION OF LOCKHEED AIRCRAFT CORPORATION

HUNTSVILLE, ALABAMA

**LOCKHEED MISSILES & SPACE COMPANY  
HUNTSVILLE RESEARCH & ENGINEERING CENTER  
HUNTSVILLE RESEARCH PARK  
4800 BRADFORD DRIVE, HUNTSVILLE, ALABAMA**

**LUNAR TARGETING STUDY  
INTERIM REPORT  
LUNAR AND PLANETARY  
EPHEMERIS TAPES**

January 1972

Contract NAS8-26578

Prepared for National Aeronautics and Space Administration  
Marshall Space Flight Center, Alabama 35812

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## FOREWORD

This report describes work accomplished by the Lockheed-Huntsville Research & Engineering Center while under contract to the Aero-Astroynamics Laboratory of Marshall Space Flight Center. This report is submitted in fulfillment of the requirements for an interim report as specified in Item 7 of Modification 2 to Contract NAS8-26578, "Lunar Targeting Study." Mr. J. D. Weiler, S&E-AERO-MFT, is the NASA Technical Coordinator for this contract.

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## SUMMARY

This report documents the Univac 1108 computer tape data formats for the Ephemeris tape generated by the Jet Propulsion Laboratory (JPL) and the Ephemeris tape used by the Quick Response Targeting Program (QRTP). Ephemeris tapes are used as the data source for the position and velocity components of those celestial bodies being considered in an integrated trajectory simulation program. The QRTP Ephemeris tape has data only for lunar mission simulations. The JPL Ephemeris tape has data for the moon and the nine planets.

The Ephemeris tapes, the data formats, the coordinate systems, and units are defined. The transformation from the mean-of-1950 coordinate system to the nearest mean Besselian year is given.

## Section I

### INTRODUCTION

A mission passes through many stages of development from the initial concept through vehicle selection and design to actual flight. The stages of mission development require the use of trajectory simulation programs of varying levels of precision. Programs used to develop the initial mission concept need not be as precise as those used to simulate preflight operational trajectories.

At some point during the development of lunar and planetary missions, the programs will require highly precise data concerning lunar and planetary position and velocity. These data can be obtained by evaluating a series of complicated general equations which describe the position and velocity of the moon and the planets. Tables of these positions and velocities for discrete times can be obtained from these equations and used with an interpolation scheme in a computer program. Such tables of the position and velocities of the planets and the moon are called "ephemerides."

Because evaluating the general equations is an involved and lengthy procedure from a computational point of view, normally a relatively simple form of the ephemerides is used by the trajectory simulation programs. These ephemerides are available in several forms. The Nautical Almanac Office of the U. S. Naval Observatory publishes a set of ephemerides for use in navigation and astronomy. The Jet Propulsion Laboratory (JPL) has produced and distributed a series of magnetic tapes containing the ephemerides.

Although the computer programs could take the ephemeris data in the form of punch cards, the cards might be dropped, shuffled, or otherwise destroyed. The use of a magnetic tape eliminates these problems, and the input is faster. The JPL has generated a series of ephemeris tapes from

the general equations of motion of moon and planets. This series of ephemeris tapes is discussed in Section 2.

For use with a lunar mission computer program, another set of ephemeris tapes was generated from the JPL tapes which are used with the Quick Response Targeting Program (QRTP). The tapes, the data they contain, and the data format are discussed in Section 3.

## Section 2

## JET PROPULSION LABORATORY EPHEMERIS TAPES

The Jet Propulsion Laboratory (JPL) Ephemeris data are recorded on three magnetic tapes. The data for the nine planets and the moon cover the years 1950 to 2000. The data are overlapped between tapes to limit, as much as possible, the requirement to mount more than one ephemeris tape for any one trajectory simulation. (See Table 1.) To improve the accuracy of the prediction of the planetary velocities over those obtained from numerical differentiation of the planetary positions, a special perturbation solution to the equations of motion for the planets was evaluated. The planetary positions and velocities are consistent with gravitational theory to high precision and are the best least-squares-fit of source position predictions.

The data in these tables are the rectangular position and velocity components for the moon and the nine planets, the longitude and obliquity nutations, and the modified second and fourth differences from the curve fit. The planetary position and velocity components are in the rectangular heliocentric equatorial coordinate system based on the 1950 epoch. The units for the planetary data are Astronomical Units (AU) and AU per mean solar day. The lunar position and velocity components are in the geocentric equatorial coordinate system based on the mean equator and equinox of the 1950 epoch. The units used for the lunar position and velocity are earth radii and earth radii per mean solar day. The data on these tapes are scaled to eliminate large numbers and to increase the precision of the data.

The longitude and obliquity are included to allow for the trajectory program using the ephemeris tapes to establish the required coordinate transformations. These are used to transform the data from the ephemeris tapes to the coordinate system used by the computer program simulating the trajectory.

The second and fourth differences from the curve fit of the ephemeris data are provided to be used by the program interpolation scheme.



JPL has plans to update the ephemeris tapes as theories from which the data were generated are improved and/or extended. The new ephemeris tapes will be generated with the same tape formats. The computer programs using the ephemeris tapes need not be modified every time JPL updates an ephemeris tape. The first two records on the JPL tapes are information records describing the data on the tape and time period the tape data covers. The record format of the tape is described in Table 2. The tape records beyond the first two are logical data records which contain the desired ephemeris data. The logical data record format is described in Table 3. The data, with the exception of the data for nutations, are in double precision. Each record contains data for an eight-day span. The data for the last day of the eight-day span on a record are repeated as the data for the first day on the next eight-day span record. This overlapping data means less tape motion and handling, as well as a simpler interpolation scheme. Within each eight-day span the lunar and nutation data is given in one-half-day steps. Data for Mercury are given in two-day steps within each eight-day span. All the other planetary data are given in four-day steps within each eight-day span. The initial day in each eight-day span is indicated at the beginning of each record in the form of the Julian date. Julian date of a calendar day may be defined as the number of days from mean noon on January 1, 4713 B.C. The Julian date is used because there are irregularities in the present calendar and a single system reduces computational labor and avoids ambiguity.

The reports and documents listed in the bibliography can provide additional information on the theoretical bases for the JPL Ephemeris tape. Documents that give a more extensive explanation of the tapes and the computer subroutines to be used with the ephemeris tapes are also included in the bibliography.

Table 1  
JPL EPHEMERIS TAPES

MSFC 1108 Source Tape No.*	JPL Tape Label	Interval Covered on Tape	
19742	DE19A	243 3280.5 (Dec 30, 1949)	244 0584.5 (Dec 29, 1969)
16573	DE19B	244 0544.5 (Nov 19, 1969)	244 5752.5 (Feb 22, 1984)
9194	DE19C	244 5612.5 (Jan 13, 1984)	245 1544.5 (Jan 1, 2000)

\*Should only be used to copy tape. Contact Billie S. Robertson, S&E-COMP-RDP, for other tape numbers and information on Ephemeris tapes.

Table 2  
JPL EPHEMERIS TAPE RECORDS

Record	Description
1st	24 BCD words written in binary. Describe general nature of information on the tape.
2nd	
1	Number of bodies on tape = 10
2	Tape format type = floating point 50
3	Initial Julian date of data on tape
4	Final Julian date of data on tape
5	Step size of logical data record = 8.0 days
6	Ten pairs of numbers. The first number in the planetary body is in increased order out from the sun, with zero used for lunar data. Second number is the step size of data given for that body.
3rd and others to End of Tape	Ephemeris data in buffered and overlapped eight-day logical records. The end points of eight-day span are repeated as first points of the succeeding eight-day records.

Table 3  
JPL EPHEMERIS TAPE LOGICAL RECORD FORMAT

Word Number	Body	Units	Word Description
1-2			Julian Date
3-4	Mercury	AU	X position at Julian Date (words 1 and 2)
5-6	Mercury	AU	Second difference for curve fit of X position component
7-8	Mercury	AU	Fourth difference for curve fit of X position component
9-10	Mercury	AU	Y position at Julian date
11-12	Mercury	AU	Second difference for curve fit of Y position component
13-14	Mercury	AU	Fourth difference for curve fit of Y position component
15-16	Mercury	AU	Z position at Julian date
17-18	Mercury	AU	Second difference for curve fit of Z position component
19-20	Mercury	AU	Fourth difference for curve fit of Z position component
21-38	Mercury	AU	Same data as in Words 3 through 20 for Julian date + 2 days
39-55	Mercury	AU	Same data as in Words 3 through 20 for Julian date + 4 days
56-74	Mercury	AU	Same data as in Words 3 through 20 for Julian date + 6 days
75-92	Mercury	AU	Same data as in Words 3 through 20 for Julian date + 8 days

Table 3 (Continued)

Word Number	Body	Units	Word Description
93-94	Mercury	AU/mean solar day	X velocity at Julian date (words 1 and 2)
95-96	Mercury	AU/mean solar day	Second difference for curve fit of X velocity component
97-98	Mercury	AU/mean solar day	Fourth difference for curve fit of X velocity component
99-104	Mercury	AU/mean solar day	Same data as in Words 93 through 98 for Y velocity component
105-110	Mercury	AU/mean solar day	Same data as in Words 93 through 98 for Z velocity component
111-128	Mercury	AU/mean solar day	Same data as in Words 93 through 110 for Julian date + 2 days
129-146	Mercury	AU/mean solar day	Same data as in Words 93 through 110 for Julian date + 4 days
147-164	Mercury	AU/mean solar day	Same data as in Words 93 through 110 for Julian date + 6 days
165-182	Mercury	AU/mean solar day	Same data as in Words 93 through 110 for Julian date + 8 days
183-200	Venus	AU	Same type position data as in Words 3 through 20 for Julian date (words 1 and 2)
201-218	Venus	AU	Same type position data as in Words 3 through 20 for Julian date + 4 days
219-236	Venus	AU	Same type position data as in Words 3 through 20 for Julian date + 8 days
237-254	Venus	AU/mean solar day	Same type velocity data as in Words 93 through 110 for Julian date (words 1 and 2)
255-272	Venus	AU/mean solar day	Same type velocity data as in Words 93 through 110 for Julian date + 4 days
273-290	Venus	AU/mean solar day	Same type velocity data as in Words 93 through 110 for Julian date + 8 days

Table 3 (Continued)

Word Number	Body	Units	Word Description
291-398	Earth Moon Barycenter	AU, AU/ mean solar day	Same type of data as in Words 183 through 290
399-506	Mars	AU, AU/ mean solar day	Same type of data as in Words 183 through 290
507-614	Jupiter	AU, AU/ mean solar day	Same type of data as in Words 183 through 290
615-722	Saturn	AU, AU/ mean solar day	Same type of data as in Words 183 through 290
723-830	Uranus	AU, AU/ mean solar day	Same type of data as in Words 183 through 290
831-938	Neptune	AU, AU/ mean solar day	Same type of data as in Words 183 through 290
939-1046	Pluto	AU, AU/ mean solar day	Same type of data as in Words 183 through 290
1047-1658	Moon	Earth Radii, Earth Radii/ mean solar day	Same type of data as in Words 183 through 290 except step size is 0.5 days
1659-1760	Nutations		$\Delta\psi, d^2\Delta\psi, d^4\Delta\psi, \Delta\epsilon, d^2\Delta\epsilon, d^4\Delta\epsilon$ repeated in 0.5 day steps for eight days
1761-1862	Nutation Rates		$\dot{\Delta\psi}, d^2\dot{\Delta\psi}, d^4\dot{\Delta\psi}, \dot{\Delta\epsilon}, d^2\dot{\Delta\epsilon}, d^4\dot{\Delta\epsilon}$ repeated in 0.5 day steps for eight days

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## Section 3

## QUICK RESPONSE TARGETING PROGRAM EPHEMERIS TAPE

The Quick Response Targeting Program (QRTP) is a series of computer programs which are used to generate and verify operational Saturn V launch vehicle targeting presettings for lunar missions. The program integrates the equations of motion of the vehicle to describe a trajectory in the earth-moon space. The four bodies whose effects are considered are: the vehicle, the earth, the moon and the sun. The program uses an ephemeris tape to obtain the position and velocity components of the earth, moon and sun with the required precision.

The portion of the QRTP that is used to simulate the coasting trajectory is the result of the evolution and development of a simple integrated coast program. The program was developed to use the lunar position and velocity data in a particular form, that of the tables from the American Ephemeris and Nautical Almanac. When the decision was made to use magnetic tape input of the ephemeris, the program in use required the data be input in the table form. The program also did not use the planetary data.

Since the JPL Ephemeris tape did not contain the data in the required form or coordinate system, the Manned Spacecraft Center in Houston generated ephemeris tapes to be used by the existing program. The JPL Ephemeris tape was used as the data source. The planetary data were not transformed to the ephemeris tape. The position and velocity of the sun and moon were transformed from the JPL heliocentric coordinate system to the nearest mean Besselian year coordinate system. The coordinate system is defined at the beginning of the mean Besselian year, which is when the right ascension of the sun is 18 hours and 40 minutes. The x-axis points to the mean vernal equinox, the z-axis is along the mean pole, and the y-axis completes the right-hand coordinate system. The mean vernal equinox, the mean pole, mean equator, and the mean ecliptic are the result of adjusting the coordinate system to compensate for the general precession. The gravitational attraction of the planets, the moon and the sun on the aspherical earth causes the axis of

rotation to precess or wobble. The effect of this wobble is defined as general precession.

The ephemeris tape covers the time period 1966 to 1999. These data have been divided into 34 record groups each of 56 records. (See Table 4.) Each record group contains the ephemeris data for a year plus 768 hours overlap at each end of the record group. This overlap reduces the tape positioning required to read the required amount of ephemeris data. Each data record group has the data based upon the nearest mean Besselian year coordinate system. (See Table 5.)

The solar and lunar data are tabulated in four-day and half-day steps, respectively. The position and velocity components are given in earth radii and earth radii per mean solar day. The precession-nutation-libration matrix for transforming the lunar data from selenocentric mean nearest Besselian year coordinates to selenographic coordinates is given in half-day steps.

The ephemeris data tape was generated from a copy of MSC Tape No. 2611. The physical tape definition is:

1. Fortran-written
2. Binary
3. 9-track
4. 800 BPI
5. Unblocked
6. Variable record format
7. Maximum block size = 3604
8. One file
9. Standard label

Two minor changes were made in the format of MSC Tape No. 2611 when it was converted from IBM 7094 to IBM 360/OS. First, all data are now double length (8 bytes). Integers occupy the first four bytes in a double-length word. Second, all file marks were removed. The tape is positioned by reading records only.

Appendix A discusses the transformation of the JPL Ephemeris data to the QRTP ephemeris tape format. The Bibliography contains those documents and reports that deal with the QRTP ephemeris tape. The theoretical bases for the data are discussed in the documents and reports listed in the Bibliography of Section 1.

Table 4  
QRTP EPHEMERIS TAPE RECORDS

<u>Record</u>	<u>Description</u>
1st	Describes the record group used to position tape
1	Number of years since 1950 epoch
2	Nearest mean Besselian year
3	Base hour of File
4	Base year for data
2nd thru 57	Ephemeris data in buffered and overlapped eight-day logical records. The end points of the eight-day span are repeated as first points of succeeding eight-day records. Each record group (records 2 thru 57) contains data for a year roughly centered about the nearest mean Besselian year.
58 thru 113 to end of tape	Ephemeris data for next year in sequence with a 768 hour overlap with the previous and following record groups.

Table 5  
QRTF EPHEMERIS LOGICAL RECORD FORMAT

<u>Word Number</u>	<u>Body</u>	<u>Units</u>	<u>Word Description</u>
1-2			Data record group number (years since 1950 epoch)
3-4			Data block number within data record group
5-6		year	Nearest mean Besselian year
7-8		hour	Hours since beginning of year (base hour)
9-10		year	Year for base hour
11-12	Sun	earth radii	X-axis position component at base hour
13-14	Sun	earth radii	Y-axis position component at base hour
15-16	Sun	earth radii	Z-axis position component at base hour
17-22	Sun	earth radii	Position vector at base hour plus four days
23-28	Sun	earth radii	Position vector at base hour plus eight days
29-34	Sun	earth radii/hour	Velocity vector at base hour
35-40	Sun	earth radii/hour	Velocity vector at base hour plus four days
41-46	Sun	earth radii/hour	Velocity vector at base hour plus eight days
47-52	Moon	earth radii	Lunar position vector at base hour
53-58	Moon	earth radii	Lunar position vector at base hour plus 0.5 day
59-64	Moon	earth radii	Lunar position vector at base hour plus 1.0 day

Table 5 (Continued)

<u>Word Number</u>	<u>Body</u>	<u>Units</u>	<u>Word Description</u>
65-70	Moon	earth radii	Lunar position vector at base hour plus 1.5 days
71-76	Moon	earth radii	Lunar position vector at base hour plus 2.0 days
77-138	Moon	earth radii	Lunar position vector at half day intervals for eight days
139-240			Lunar velocity vectors at half day intervals from base hour for eight days
241-556			The precession, nutation, and libration matrix for eight days in half day intervals from base hour angle

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Appendix

JET PROPULSION LABORATORY TO QUICK  
RESPONSE TARGETING PROGRAM  
EPHEMERIS TRANSFORMATION

### Appendix

The data from the JPL Ephemeris tapes were used to generate the QRTP tapes. Because the JPL and the QRTP Ephemeris tapes use different reference coordinate systems, the data must be transformed from the mean-of-1950 coordinate system (JPL tapes) to the mean-nearest-Besselian year coordinate system (QRTP tape). These coordinate systems are defined in Figs. A-1 and A-2, respectively. The two coordinate systems differ due to the precession and nutation of the earth's rotational axis.

The geometry of precession is represented by the parameters  $\zeta_0$ ,  $z$  and  $\theta$  in Fig. A-3. The analytical expressions for  $\zeta_0$ ,  $z$  and  $\theta$  as defined in Ref. A-1 are as follows:

$$\zeta_0 = 2304.''997T + 0.''302T^2 + 0.''0179T^3$$

$$z = 2304.''997T + 1.''093T^2 + 0.''0192T^3$$

$$\theta = 2004.''298T - 0.''426T^2 - 0.''0416T^3$$

with  $T$  the number of Julian centuries of 36,525 days past the epoch 1950.0.

The general precession of the Earth's equator and the consequent retrograde motion of the equinox on the ecliptic may be represented by the rotation matrix:

$$\begin{pmatrix} X' \\ Y' \\ Z' \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

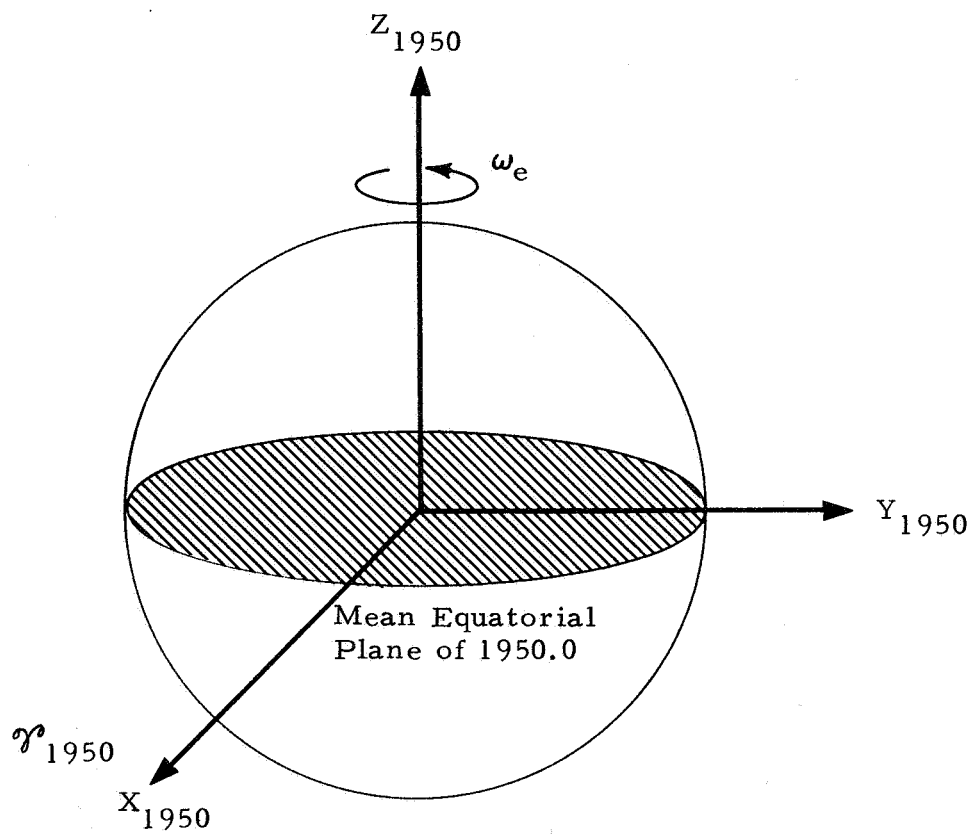


Fig. A-1 - Mean-of-1950.0 Coordinate System

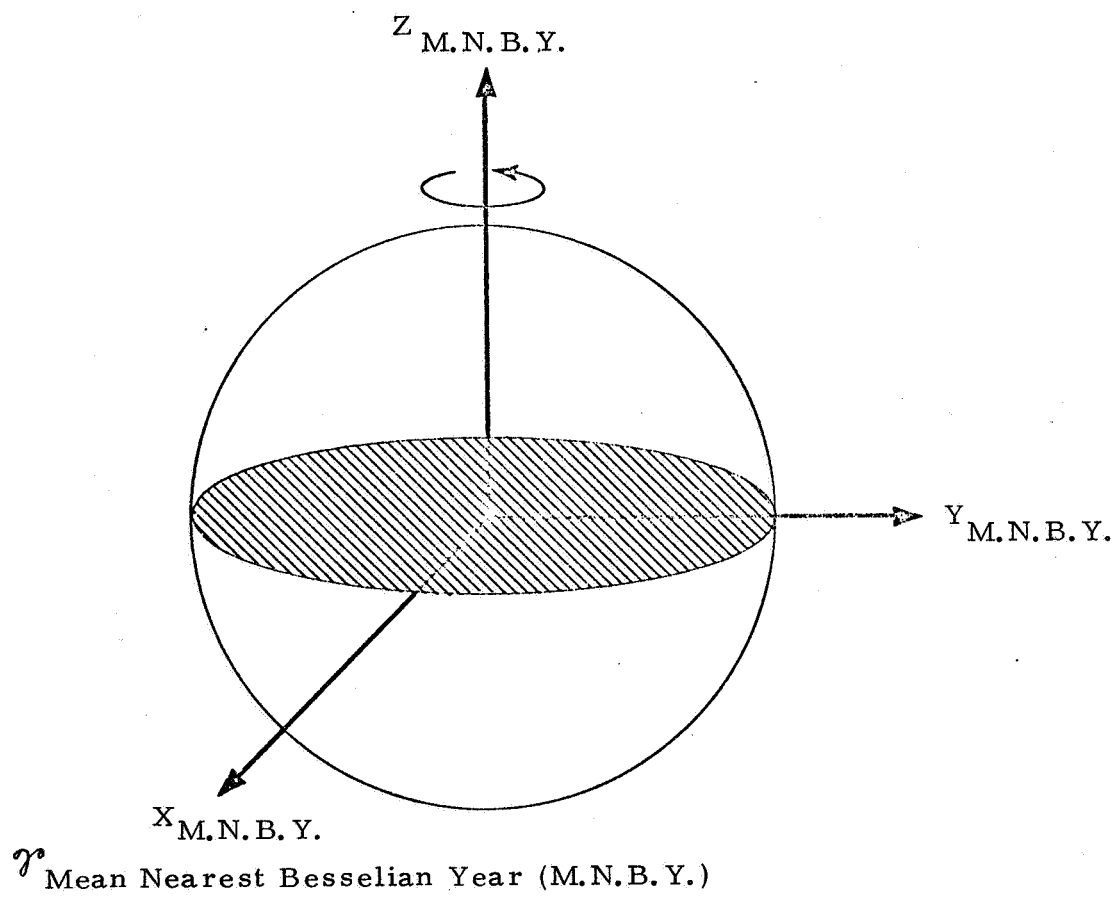


Fig. A-2 - Mean Nearest Besselian Year Coordinate System

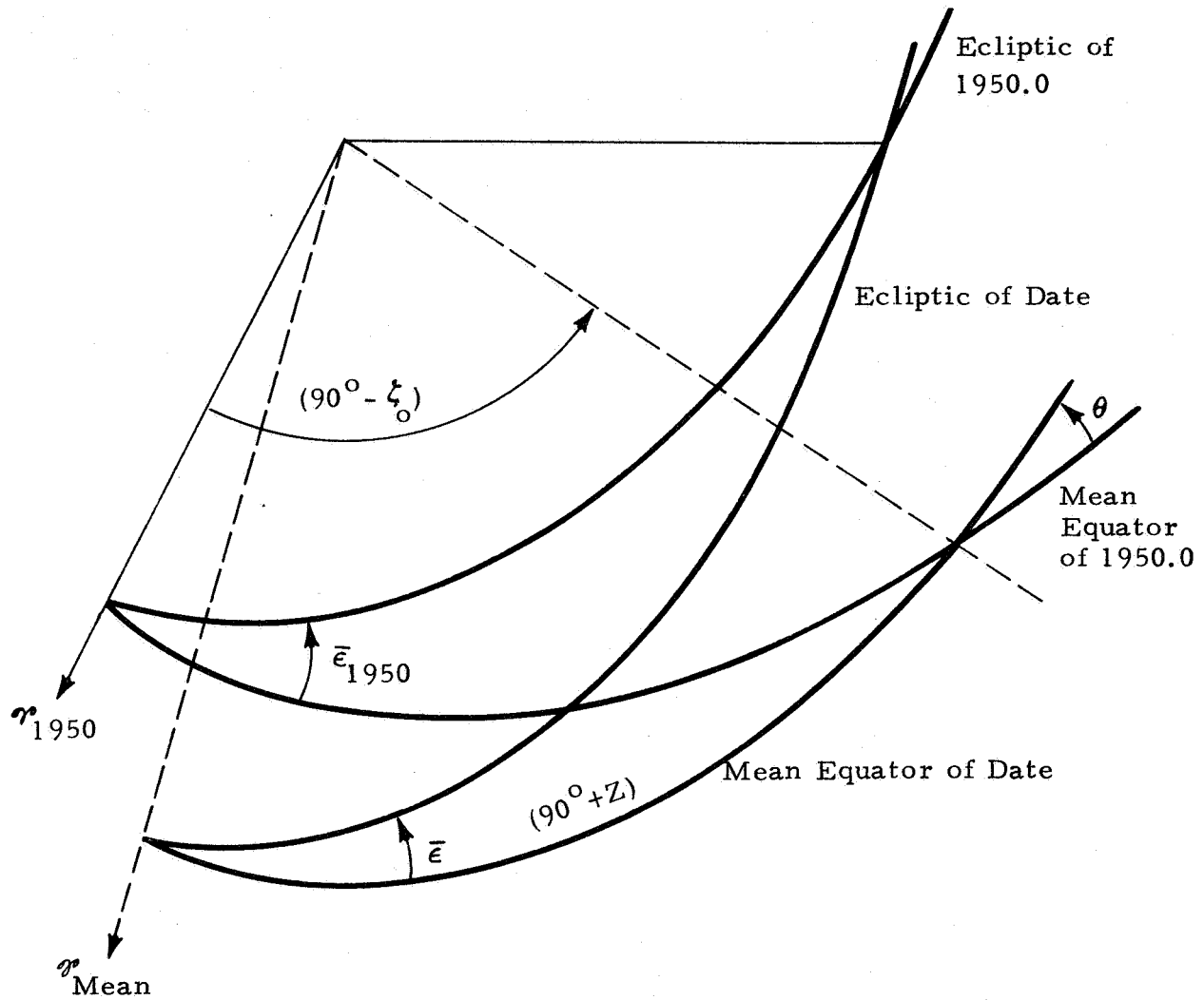


Fig. A-3 - Geometry of Precession

where

$X, Y, Z$  = coordinates in the mean equator and equinox of 1950

$X', Y', Z'$  = coordinates in the mean equator and equinox of date

and

$$\begin{aligned} a_{11} &= -\sin \zeta_0 \sin z + \cos \zeta_0 \cos z \cos \theta \\ a_{12} &= -\cos \zeta_0 \sin z - \sin \zeta_0 \cos z \cos \theta \\ a_{13} &= -\cos z \sin \theta \\ a_{21} &= \sin \zeta_0 \cos z + \cos \zeta_0 \sin z \cos \theta \\ a_{22} &= \cos \zeta_0 \cos z - \sin \zeta_0 \sin z \cos \theta \\ a_{23} &= -\sin z \sin \theta \\ a_{31} &= \cos \zeta_0 \sin \theta \\ a_{32} &= -\sin \zeta_0 \sin \theta \\ a_{33} &= \cos \theta \end{aligned}$$

The geometry of nutation of the Earth about the precessing mean equator is represented by the parameters  $\delta\psi$  and  $\delta\epsilon$  as shown in Fig. A-4. The values of  $\delta\psi$  and  $\delta\epsilon$  are available from the JPL Ephemeris tapes. The nutation of the Earth's equator may be represented by the rotation matrix:

$$\begin{pmatrix} x'' \\ y'' \\ z'' \end{pmatrix} = \begin{pmatrix} N_{11} & N_{12} & N_{13} \\ N_{21} & N_{22} & N_{23} \\ N_{31} & N_{32} & N_{33} \end{pmatrix} \begin{pmatrix} x' \\ y' \\ z' \end{pmatrix}$$

where

$x', y', z'$  = coordinates in the mean equator and equinox of date

$x'', y'', z''$  = coordinates in the true equator and equinox of date

and

$$\begin{aligned} N_{11} &= \cos \delta\psi \\ N_{12} &= \sin \delta\psi \cos \bar{\epsilon} \\ N_{13} &= \sin \delta\psi \sin \bar{\epsilon} \end{aligned}$$

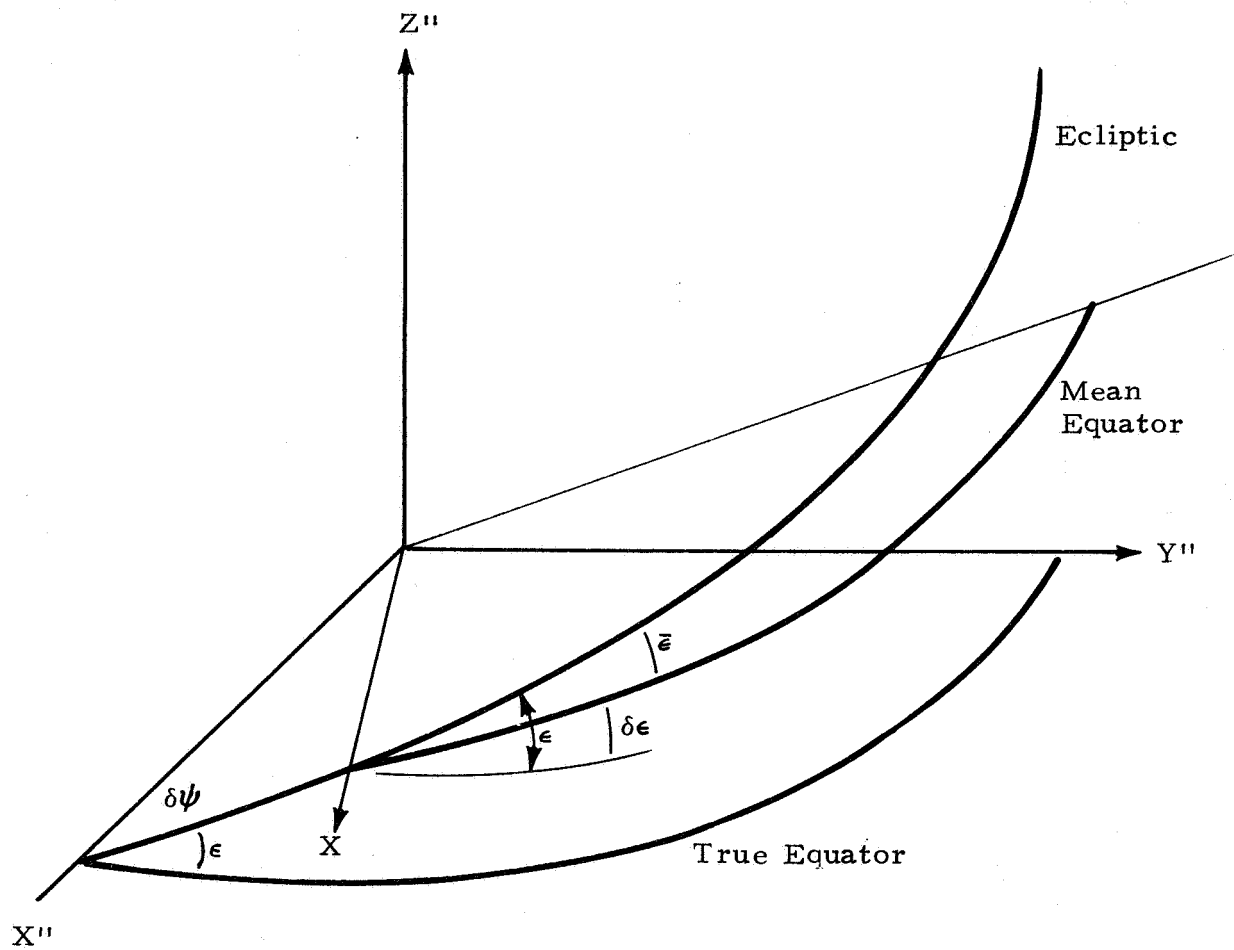


Fig. A-4 - Geometry of Nutation

$$N_{21} = \sin \delta \psi \cos \epsilon$$

$$N_{22} = \cos \delta \psi \cos \epsilon \cos \bar{\epsilon} + \sin \epsilon \sin \bar{\epsilon}$$

$$N_{23} = \cos \delta \psi \cos \epsilon \sin \bar{\epsilon} - \sin \epsilon \cos \bar{\epsilon}$$

$$N_{31} = \sin \delta \psi \sin \epsilon$$

$$N_{32} = \cos \delta \psi \sin \epsilon \cos \bar{\epsilon} - \cos \epsilon \sin \bar{\epsilon}$$

$$N_{33} = \cos \delta \psi \sin \epsilon \sin \bar{\epsilon} + \cos \epsilon \cos \bar{\epsilon}$$

$$\begin{aligned} \bar{\epsilon} = 23.4457587 - 0.01309404T - 0.0088 \times 10^{-4} T^2 \\ + 0.0050 \times 10^{-4} T^3 \end{aligned}$$

$$\epsilon = \bar{\epsilon} + \delta \epsilon$$

The [a] and [N] matrix are then multiplied to get a single matrix [Na] to rotate from the mean equator and equinox of 1950.0 to the true equator and equinox of date.

To allow vectors in the 1950.0 system to be expressed relative to the moon's true equator, a matrix to describe the libration of the moon must be generated. The geometry relation of the Earth's true equator and equinox to the moon's true equator is shown in Fig. A-5. The analytical expression for the angles defined in Fig. A-5 were obtained from Ref. A-1. These expressions are:

$$\cos(i) = \cos(\Omega + \sigma + \delta\psi) \sin \epsilon \sin(I + \rho) + \cos \epsilon \cos(I + \rho), \quad 0 < i < 90^\circ$$

$$\sin \Omega' = -\sin(\Omega + \sigma + \delta\psi) \sin(I + \rho) \csc(i), \quad -90^\circ < \Omega' < 90^\circ$$

$$\sin \Delta = -\sin(\Omega + \sigma + \delta\psi) \sin \epsilon \csc(i)$$

$$\cos \Delta = -\sin(\Omega + \sigma + \delta\psi) \sin \Omega' \cos \epsilon - \cos(\Omega + \sigma + \delta\psi) \cos \Omega', \quad 0 \leq \Delta < 360^\circ$$

$$\Lambda = \Delta + (\zeta + \tau) - (\Omega + \sigma)$$



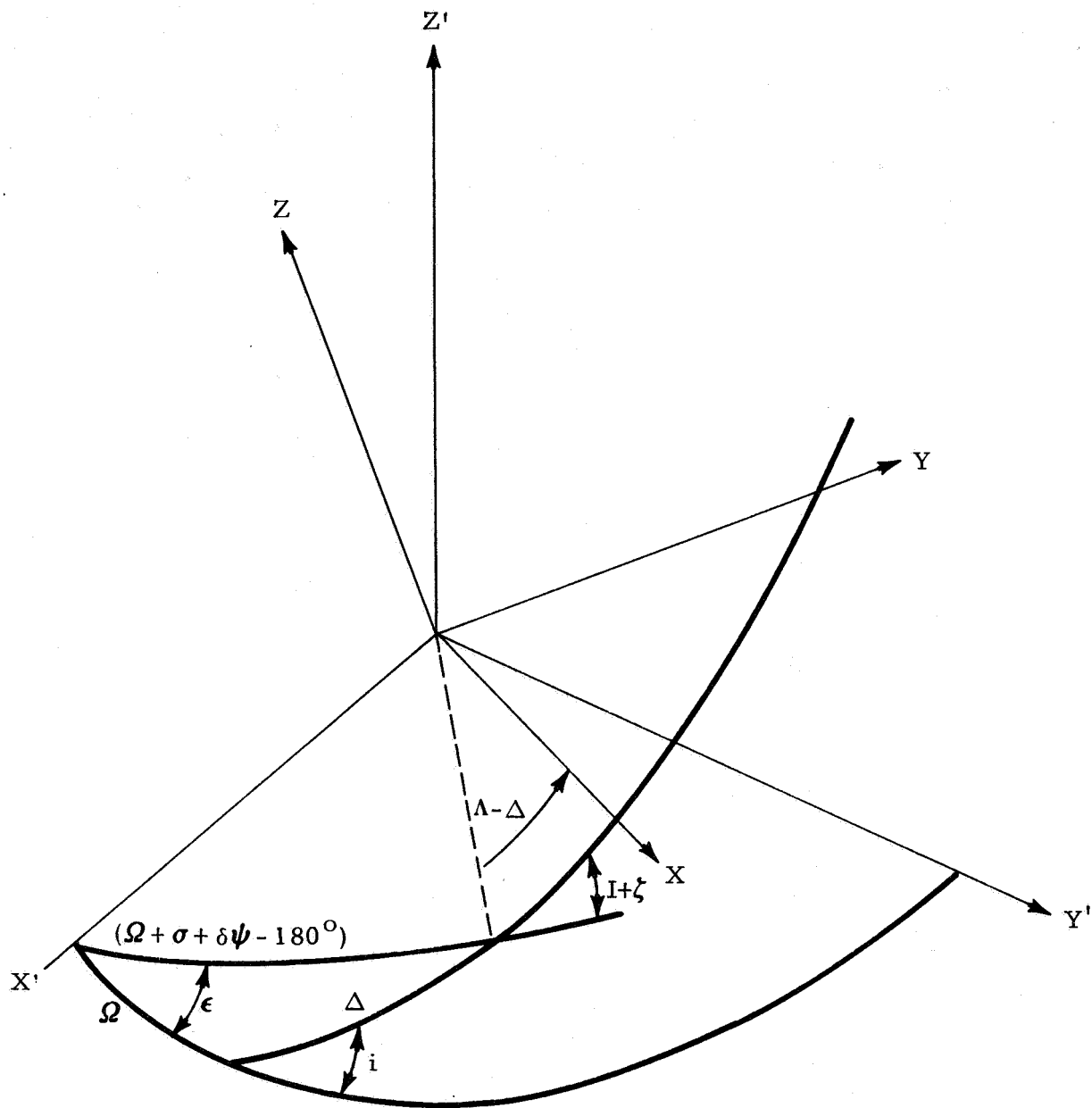


Fig. A-5 - Geometry of the Lunar Librations

where

$$\sigma \sin I = -0.0302777 \sin g + 0.0102777 \sin(g + 2\omega) \\ - 0.00305555 \sin(2g + 2\omega)$$

$$\tau = -0.003333 \sin g + 0.0163888 \sin g' + 0.005 \sin 2\omega$$

$$\rho = -0.0297222 \cos g + 0.0102777 \cos(g + 2\omega) \\ - 0.00305555 \cos(2g + 2\omega)$$

$$I = 1.535$$

$$g = 215.54013 + 13.064992 d$$

$$g' = 358.009067 + 0.9856005 d$$

$$\omega = 196.745632 + 0.1643586 d$$

$$\Delta\epsilon = 25.5844 \times 10^{-4} \cos \Omega - 0.2511 \times 10^{-4} \cos 2\Omega \\ + 1.5336 \times 10^{-4} \cos 2L + 0.0666 \times 10^{-4} \cos(3L - \Gamma) \\ - 0.0258 \times 10^{-4} \cos(L + \Gamma) - 0.0183 \times 10^{-4} \cos(2L - \Omega) \\ - 0.0067 \times 10^{-4} \cos(2\Gamma' - \Omega)$$

$$d\epsilon = 0.2456 \times 10^{-4} \cos 2\zeta + 0.0508 \times 10^{-4} \cos(2\zeta - \Omega) \\ + 0.0369 \times 10^{-4} \cos(3\zeta - \Gamma') - 0.0139 \times 10^{-4} \cos(\zeta + \Gamma') \\ - 0.0086 \times 10^{-4} \cos(\zeta - \Gamma' + \Omega) + 0.0083 \times 10^{-4} \cos(\zeta - \Gamma' - \Omega) \\ + 0.0061 \times 10^{-4} \cos(3\zeta + \Gamma' - 2L) + 0.0064 \times 10^{-4} \cos(3\zeta - \Gamma' - \Omega)$$

$$\begin{aligned}\Delta\psi = & - (47.8927 + 0.0482T) \times 10^{-4} \sin\Omega + 0.5800 \times 10^{-4} \sin 2\Omega \\ & - 3.5361 \times 10^{-4} \sin 2L - 0.1378 \times 10^{-4} \sin(3L - \Gamma) \\ & + 0.0594 \times 10^{-4} \sin(L + \Gamma) + 0.0344 \times 10^{-4} \sin(2L - \Omega) \\ & + 0.0125 \times 10^{-4} \sin(2\Gamma' - \Omega) + 0.3500 \times 10^{-4} \sin(L - \Gamma) \\ & + 0.0125 \times 10^{-4} \sin(2L - 2\Gamma')\end{aligned}$$

$$\begin{aligned}d\psi = & - 0.5658 \times 10^{-4} \sin 2\zeta - 0.0950 \times 10^{-4} \sin(2\zeta - \Omega) \\ & - 0.0725 \times 10^{-4} \sin(3\zeta - \Gamma') + 0.0317 \times 10^{-4} \sin(\zeta + \Gamma') \\ & + 0.0161 \times 10^{-4} \sin(\zeta - \Gamma' + \Omega) + 0.0158 \times 10^{-4} \sin(\zeta - \Gamma' - \Omega) \\ & - 0.0144 \times 10^{-4} \sin(3\zeta + \Gamma' - 2L) - 0.0122 \times 10^{-4} \sin(3\zeta - \Gamma' - \Omega) \\ & + 0.1875 \times 10^{-4} \sin(\zeta - \Gamma') + 0.0078 \times 10^{-4} \sin(2\zeta - 2\Gamma') \\ & + 0.0414 \times 10^{-4} \sin(\zeta + \Gamma' - 2L) + 0.0167 \times 10^{-4} \sin(2\zeta - 2L) \\ & - 0.0089 \times 10^{-4} \sin(4\zeta - 2L)\end{aligned}$$

$$\begin{aligned}\Omega = & 12.1127902 - 0.0529539222d + 20.795 \times 10^{-4}T \\ & + 20.81 \times 10^{-4}T^2 + 0.02 \times 10^{-4}T^3\end{aligned}$$

$$\begin{aligned}\zeta = & 64.37545167 + 13.1763965268d - 11.31575 \times 10^{-4}T \\ & - 11.3015 \times 10^{-4}T^2 + 0.019 \times 10^{-4}T^3\end{aligned}$$

$$\begin{aligned}\Gamma' = & 208.8439877 + 0.1114040803d - 0.010334T \\ & - 0.010343T^2 - 0.12 \times 10^{-4}T^3\end{aligned}$$

$$\begin{aligned}L = & 280.08121009 + 0.9856473354d + 3.03 \times 10^{-4}T \\ & + 3.03 \times 10^{-4}T^2\end{aligned}$$

$$\begin{aligned}\Gamma = & 282.08053028 + 0.470684 \times 10^{-4}d + 4.5525 \times 10^{-4}T \\ & + 4.575 \times 10^{-4}T^2 + 0.03 \times 10^{-4}T^3\end{aligned}$$

d = number of days past the epoch 1950.0

T = number of Julian centuries of 36,525 days past the epoch 1950.0

The libration of the moon may be represented by the rotation matrix:

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix} \begin{pmatrix} X'' \\ Y'' \\ Z'' \end{pmatrix}$$

where

$X'', Y'', Z''$  = coordinates in the Earth's true equator and equinox of date

$x, y, z$  = coordinates in the moon's true equator

and

$$b_{11} = \cos\Lambda \cos\Omega' - \sin\Lambda \sin\Omega' \cos(i)$$

$$b_{12} = \cos\Lambda \sin\Omega' + \sin\Lambda \cos\Omega' \cos(i)$$

$$b_{13} = \sin\Lambda \sin(i)$$

$$b_{21} = -\sin\Lambda \cos\Omega' - \cos\Lambda \sin\Omega' \cos(i)$$

$$b_{22} = -\sin\Lambda \sin\Omega' + \cos\Lambda \cos\Omega' \cos(i)$$

$$b_{23} = \cos\Lambda \sin(i)$$

$$b_{31} = \sin\Omega' \sin(i)$$

$$b_{32} = -\cos\Omega' \sin(i)$$

$$b_{33} = \cos(i)$$

The matrix product of the libration, nutation and precession matrix is used to transform position vectors from the mean of 1950.0 system to the true lunar equatorial system.

To obtain velocity transformations, the above formulas are differentiated and the approximation is made that

$$\dot{N} = \dot{A} = 0$$

Thus

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = MNA \begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix} + \dot{M}NA \begin{pmatrix} X \\ Y \\ Z \end{pmatrix}$$

and for the inverse transformation

$$\begin{pmatrix} \dot{X} \\ \dot{Y} \\ \dot{Z} \end{pmatrix} = (MNA)' \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} + (\dot{M}NA)' \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

In computing  $\dot{M}$  the rates for the slowly varying angles  $\Omega'$  and  $i$  are taken to be zero.

$$\dot{M} = (\dot{M}_{ij})$$

where

$$\dot{M}_{11} = (-\sin\Lambda \cos\Omega' - \cos\Lambda \sin\Omega' \cos(i)) \dot{\Lambda}$$

$$\dot{M}_{12} = (-\sin\Lambda \sin\Omega' + \cos\Lambda \cos\Omega' \cos(i)) \dot{\Lambda}$$

$$\dot{M}_{13} = (\cos\Lambda \sin(i)) \dot{\Lambda}$$

$$\dot{M}_{21} = (-\cos\Lambda \cos\Omega' + \sin\Lambda \sin\Omega' \cos(i)) \dot{\Lambda}$$

$$\dot{M}_{22} = (-\cos\Lambda \sin\Omega' - \sin\Lambda \cos\Omega' \cos(i)) \dot{\Lambda}$$

$$\dot{M}_{23} = (-\sin\Lambda \sin(i)) \dot{\Lambda}$$

$$\dot{M}_{31} = 0$$

$$\dot{M}_{32} = 0$$

$$\dot{M}_{33} = 0$$

From the formula

$$\Lambda = \Delta + (\zeta + \tau) - (\Omega + \sigma)$$

obtain

$$\dot{\Lambda} = \dot{\Delta} + \dot{\zeta} + \dot{\tau} - \dot{\Omega} - \dot{\sigma}$$

The adopted numerical expressions for the rates are

$$\dot{\Delta} = \frac{-\cos(\Omega + \sigma + \delta\psi) \sin(\dot{\Omega} + \dot{\sigma})}{\sin(i) \cos \Delta}$$

$$\dot{\zeta} = 0.266170762 \times 10^{-5} - 0.12499171 \times 10^{-13} \text{ T rad/sec}$$

$$\dot{\Omega} = -0.1069698435 \times 10^{-7} + 0.23015329 \times 10^{-13} \text{ T rad/sec}$$

$$\begin{aligned} \dot{\tau} = & -0.1535272946 \times 10^{-9} \cos g \\ & + 0.569494067 \times 10^{-10} \cos g' \\ & + 0.579473484 \times 10^{-11} \cos 2\omega \text{ rad/sec} \end{aligned}$$

$$\begin{aligned} \dot{\sigma} = & -0.520642191 \times 10^{-7} \cos g \\ & + 0.1811774451 \times 10^{-7} \cos(g + 2\omega) \\ & - 0.1064057858 \times 10^{-7} \cos(2\omega + 2g) \text{ rad/sec} \end{aligned}$$

REFERENCE

- A-1. Holdridge, D. B., "Space Trajectories Program for the IBM 7090 Computer," JPL TR 32-223, 2 March 1962.